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Variability of the radiant heating rate at the border between  
water masses with different absorption properties – a  
theoretical prediction

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**Abstract**

The spatial distribution of the radiant heating rate (RHR) in an area of seawater absorption change was investigated by means of a theoretical model. It was found that a sufficiently high gradient of the absorption coefficient in a horizontal direction could contribute significantly to the water's warming. When combined with an absorption increase in the vertical direction, it raised the local RHR value to about one and a half times its value in the adjacent area. Moreover, when solar energy propagated through the water of higher absorption and entered the area of lower absorption, the horizontal component of the absorption gradient resulted in a lowering of the RHR value. The highest obtained spatial differences between the radiant heating rate values was of the order of  $10^{-2}$  °C per hour under the conditions of a cloudless sky with the sun zenith angle at 45° and sea surface irradiance value of nearly 700 W m<sup>-2</sup>.

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## INTRODUCTION

The radiant heating rate (RHR) describes seawater temperature increase per unit time resulting from the absorption of solar radiative energy. In the uppermost levels of the ocean, heating is governed by the strong absorption of near-infrared radiative energy by pure water. At greater depths, where long wave radiation is completely removed, optically active seawater constituents start to play a significant role in light energy absorption (Siegel et al. 1995, Ohlmann et al. 1998, Ohlmann et al. 2000). High concentrations of phytoplankton can locally intensify the warming in the surface layer (Ramp et al. 1991, Kahru et al. 1993).

The atmospheric illumination conditions, sea surface state and visible optical attenuation are the most important factors that influence the RHR temporal and spatial variability (Ohlmann et al. 2000). In a homogenous sea, RHR monotonically decreases as a function of depth due to the irradiance decrease (Morel and Antoine 1994, Ohlmann et al. 2000). In several papers, however, it was deduced that in spite of the loss of radiative energy down the water column, an increase in attenuating material concentration at a certain depth could lead to a local increase in RHR (Stavn 1982, Lewis et al. 1983, Czyszek 1991). The direct reason for such an inhomogeneity could be the phytoplankton vertical distribution (Lewis et al. 1983). It can be expected that in coastal waters other attenuating substances, for example dissolved organic matter, also play an important role in water heating. Estimation based on the measured profiles of algal concentrations in the ocean (Lewis et al. 1983) and simulated profiles of optical properties for the Baltic Sea conditions (Czyszek 1991) showed that RHR increase was on the order of  $10^{-4}$  -  $10^{-2}$  °C h<sup>-1</sup>, depending on the magnitude of the irradiance at the sea surface. Possible consequences of non-homogenous vertical RHR distribution can be convective and double-diffusion processes caused by temperature inversion, deepening of the seasonal thermocline or the maintenance of density gradients (Lewis et al. 1983). As of yet, there is no convincing evidence of the existence of such a phenomenon and the role it would play in natural waters. Undoubtedly, the difficulties in unequivocally demonstrating this phenomenon are caused by small differences in temperature fields generated by optical variability. In fact, there are many other active dynamic processes that can mask this type of temperature effect.

Horizontal variability in the radiant heating rate can also influence seawater dynamics. For example, surface water in an area of high phytoplankton biomass is heated more than the water outside of it. This causes a horizontal pressure gradient across the front between the waters of different optical properties,

which can induce specific circulation around phytoplankton-rich water (Edwards et al. 2001, Edwards et al. 2004).

In this study, the RHR spatial distribution was investigated with the use of a simple, two-dimensional theoretical model. The objective of this work was not to evaluate the role of particular seawater constituents in forming the RHR field or to accurately simulate real, marine conditions. The primary purpose was to show that the occurrence of a horizontal gradient in optical properties could directly change the water's warming. This was achieved by considering the variability of seawater absorption as an example.

#### DESCRIPTION OF THE METHOD

The RHR value at a given point in the sea can be determined when the dependencies of spectral scalar irradiance ( $E_o$  [ $\text{W m}^{-2} \text{nm}^{-1}$ ]) and absorption coefficient ( $a$  [ $\text{m}^{-1}$ ]) on spatial coordinates ( $x_i$  [m]) are known. Because solar energy arrives at the sea in the 300-4000 nm wavelength band it can be expressed in a following way (Czyszek 1991)

$$\frac{\partial T(x_i)}{\partial t} = \frac{\int_{300\text{nm}}^{4000\text{nm}} a(x_i, \lambda) E_o(x_i, \lambda) d\lambda}{\rho c_p} \quad (1)$$

where the RHR is expressed in [ $^{\circ}\text{C s}^{-1}$ ],  $c_p$  [ $\text{J kg}^{-1} \text{ }^{\circ}\text{C}^{-1}$ ] is the thermal capacity and  $\rho$  [ $\text{kg m}^{-3}$ ] is the density of seawater.

The total seawater absorption coefficient is the sum of the absorption coefficients of optically active constituents ( $a_c$ ) and pure water itself ( $a_w$ ). The absorption of pure water dominates in the long wavelength band, whereas in the visible band, suspended particulate and dissolved organic matter also significantly contribute to the absorption process (Jerlov 1976, Dera 2003). Therefore, the integral in Eq.(1) was divided into the integrals over visible and longwave parts of the spectrum, i.e. 300-700 and 700-4000 nm, respectively. In order to obtain the final result, these two integrals were added.

The visible irradiance values in the water body were derived from a solution to Gershun's stationary equation in source-free water. The equation relates the divergence of the irradiance vector, i.e. the light energy absorbed per cubic metre in seawater, to the absorptive properties of seawater (Jerlov 1976, Dera 2003)

$$\nabla \vec{E}(x_i, \lambda) = -a(x_i, \lambda) E_o(x_i, \lambda) \quad (2)$$

where  $\vec{E} = [E_i]$  is the irradiance vector [ $\text{W m}^{-2} \text{nm}^{-1}$ ].

The irradiance vector was expressed further as  $\vec{E} = [\mu_i E_o]$  where  $\mu_i$  were the cosines of angles with respect to all axes of a unit vector pointing in the average propagation direction of the entire light field. For each wavelength, the absorption coefficient of a hypothetical optically active constituent was modelled with the use of a hyperbolic function

$$a_c(x, z) = \pm f \tanh[d(x + gz)] + m. \quad (3)$$

This function describes a general situation where two water masses with different absorption capabilities make contact along an inclined plane. The top and bottom signs were used for the absorption increase or decrease towards positive  $x$ , respectively. Values of  $m - f$  and  $m + f$  ( $f$  positive) were the lower and higher absorption coefficients,  $a_{c1}$  and  $a_{c2}$ , respectively. The coefficient values changed across the mixing zone, whose width was determined by the parameter  $d$ . For example,  $d = 3 \text{ m}^{-1}$  meant that the absorption change took place almost entirely within a distance of approximately 1.3 m. The higher the value of  $d$ , the faster the absorption varied. The centre of the mixing zone, characterised by the value of the absorption coefficient equal to  $m$ , was located on the plane  $0 = x + gz$ . For simplicity, the plane intersected the sea surface at the  $y$ -axis. The tangent of the inclination angle of the plane with respect to the  $z$ -axis (oriented upwards) was positive ( $-g$ ). In order to solve Gershun's equation for scalar irradiance, it was assumed that the mean direction of energy propagation, lying on the  $xz$  plane, was constant, i.e.  $\mu_x$  and  $\mu_z$  were constant. Under the above assumptions, the equation took the form

$$\frac{\partial E_o}{\partial x} \mu_x + \frac{\partial E_o}{\partial z} \mu_z = -\{\pm f \tanh[d(x + gz)] + m + a_w\} E_o. \quad (4)$$

In setting the border conditions, irradiance reflectance caused by light scattering in the water (ratio of upwelling to downwelling irradiances) as well as direct reflectance from both sides of the sea surface were neglected. For  $E_o(x, z = 0) = E_{os}$ , with  $E_{os}$  as the atmospheric irradiance at the sea surface, the following particular solution was obtained

$$E_o = E_{os} \exp\left[-\frac{(m + a_w)}{\mu_z} z\right] \left\{ \frac{\cosh\left[d\mu_x \left(\frac{x}{\mu_x} - \frac{z}{\mu_z}\right)\right]}{\cosh[d(x + gz)]}\right\}^{\frac{f}{d\mu_x + dg\mu_z}} \quad (5)$$

For longer wavelengths, the total irradiance  $E_z(z, 700-4000\text{nm})$  [ $\text{W m}^{-2}$ ] was parameterized with the use of a three-component function (Czyszek 1988)

$$E_z(z, 700-4000\text{nm}) = E_{zs}(700-4000\text{nm}) \sum_{k=1}^3 \alpha_k \exp(-\beta_k z) \quad (6)$$

where  $E_{zs}(700-4000\text{nm})$  is the irradiance at the sea surface and the values of the coefficients  $\alpha$  (dimensionless) and  $\beta$  [ $\text{m}^{-1}$ ] are listed below:

$$\alpha_1=0.234, \alpha_2=0.177, \alpha_3=0.589, \beta_1=2.239, \beta_2=4.624, \beta_3=32.275.$$

The above dependency was used to estimate water heating resulting only from pure water absorption of radiant energy according to Eq. (2).

Finally, RHR was calculated from the following formula

$$RHR = \frac{\int_{300\text{nm}}^{700\text{nm}} a(x, z, \lambda) E_o(x, z, \lambda) d\lambda - \frac{dE_z(z, 700-4000\text{nm})}{dz}}{\rho c_p} \quad (7)$$

The integral over the visible part of the spectrum in Eq. (7) was evaluated numerically with 10 nm steps.

For comparison, the RHR was also determined in a horizontally stratified sea, where the border between water masses coincided with a horizontal plane at a given depth  $z_b$ . The seawater absorption coefficient increase at greater depths was approximated by the function  $a(z) = -f \tanh[d(z - z_b)] + m + a_w$  with parameters of the same meaning as in Eq. (3) but related to the vertical direction. In this case, the scalar irradiance varied with depth in the following way

$$E_o = E_{os} \exp\left[-\frac{(m + a_w)}{\mu_z} z\right] \left\{ \frac{\cosh[d(z - z_b)]}{\cosh(dz_b)}\right\}^{\frac{fd}{\mu_z}} \quad (8)$$

The RHR was then determined in the same manner as described earlier.

External illumination conditions were chosen for a cloudless sky with the sun zenith angle at  $45^\circ$ . The solar radiant energy focused around this direction. After refraction of the sun's rays at the sea surface, the underwater zenith angle, according to Snell's law, was approximately  $32^\circ$  and  $\mu_z = -0.85$ . Due to the very pronounced peak of solar energy in a single direction, the average cosine with respect to the  $x$ -axis was approximated as  $\mu_x \approx \sqrt{1 - \mu_z^2} = 0.53$  for all visible wavelengths. The spectral distribution of the solar energy ( $E_{zs}(\lambda)$ ) at the sea surface was determined with the use of a model for Baltic Sea atmospheric conditions on a July day (Krężel 1994) (Fig. 1a). The total irradiance was equal to the value  $688 \text{ Wm}^{-2}$  and the irradiance in the visible band constituted almost half of this value (49.4%). The chosen absorption spectra of seawater components ( $a_{c1} < a_{c2}$ ) were typical of coastal waters, which are abundant in dissolved organic matter as well as detritus particles

(Roesler and Perry 1989) (Fig. 1b).

The parameters in  $a(x, z)$ ,  $a(z)$  functions were then calculated as  $f = 0.5(a_{c2} - a_{c1})$  and  $m = 0.5(a_{c2} + a_{c1})$ . The absolute values of the partial and the highest directional derivatives of the absorption coefficient at the centre of the mixing zone were

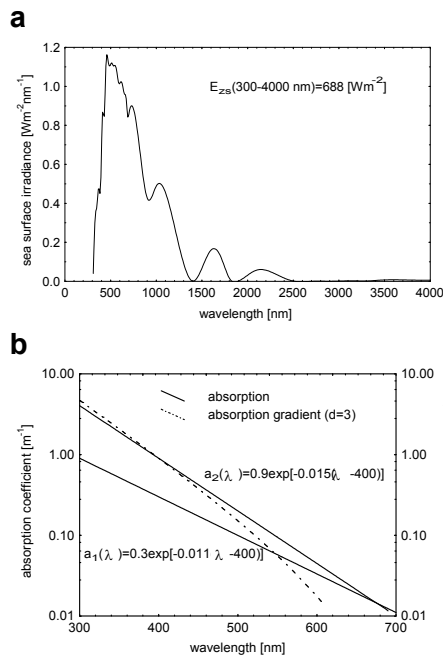
$$\left| \frac{\partial a}{\partial x} \right| = 0.5(a_{c2} - a_{c1})d, \quad \left| \frac{\partial a}{\partial z} \right| = 0.5(a_{c2} - a_{c1})dg,$$

$$|\text{grad}(a)| = 0.5(a_{c2} - a_{c1})d\sqrt{(1 + g^2)},$$

and the derivative with respect to depth in the horizontally stratified

$$\text{sea was } \left| \frac{da}{dz} \right| = 0.5(a_{c2} - a_{c1})d$$

For the highest  $d = 3 \text{ [m}^{-1}\text{]}$  used in the calculations, the absorption horizontal derivative in the visible band ranged in value from about  $1 \text{ m}^{-2}$  at  $400 \text{ nm}$  to almost zero at long wavelengths (Fig. 1b). Water thermal capacity was taken as



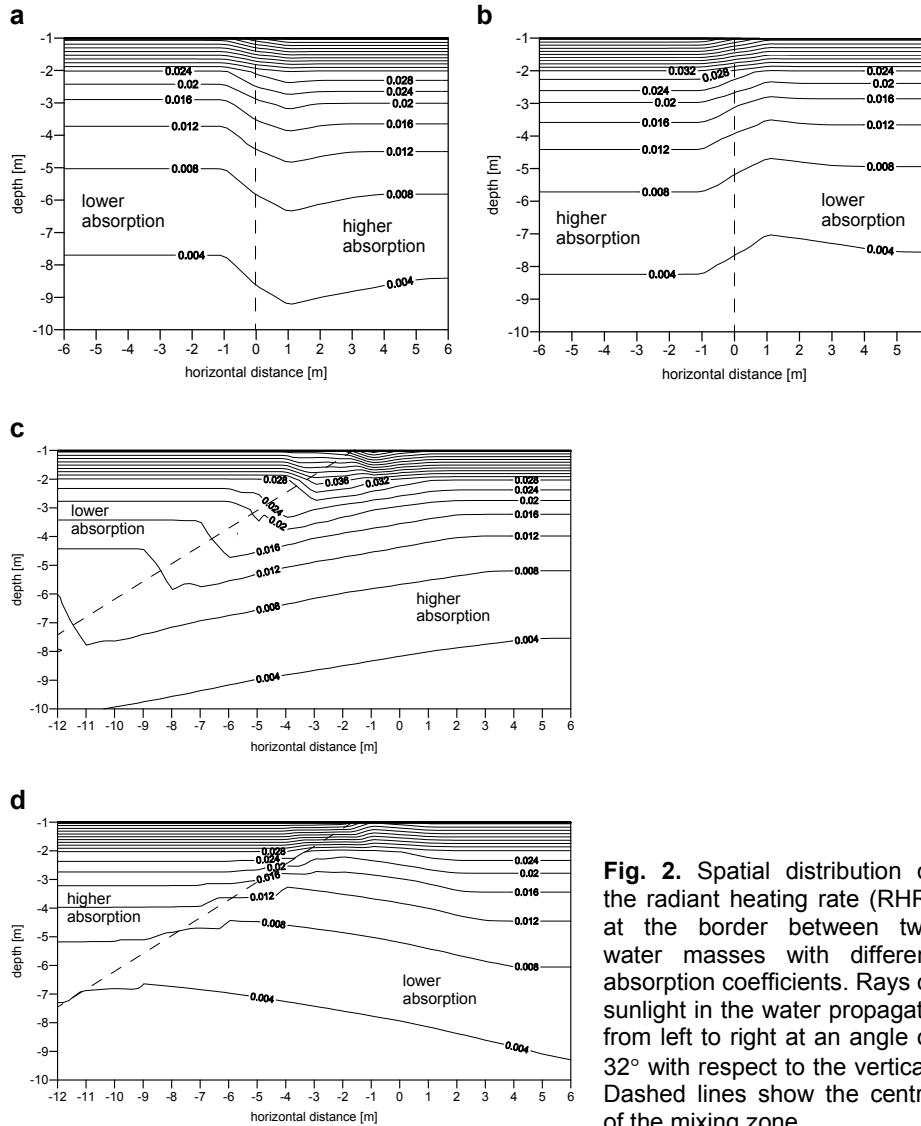
**Fig. 1.** (a) Spectral functions of atmospheric irradiance at the sea surface, and (b) absorption properties of the seawater component.

4167 J kg<sup>-1</sup> °C<sup>-1</sup> and water density as in Baltic Sea surface waters – 1010 kg m<sup>-3</sup>.

## RESULTS

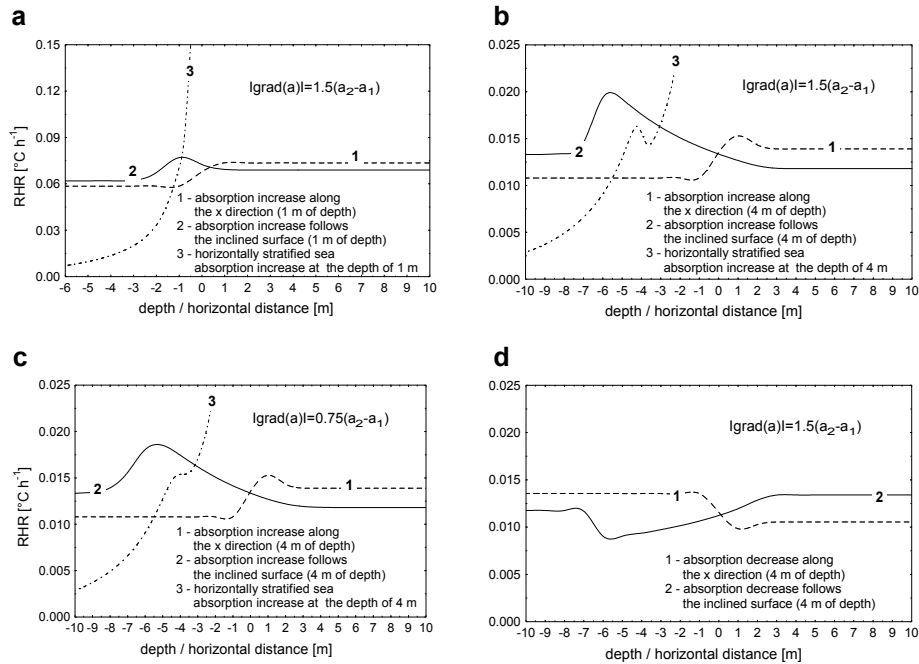
In order to check how the horizontal absorption gradient influenced the RHR spatial distribution, the situation was examined when waters were mixing across the vertical plane. In this case,  $g$  was equal to zero in Eqs. 3 and 5. The first computations were carried out with  $d=3$  [m<sup>-1</sup>].

When solar energy first penetrated the clearer water, a zone of higher RHR values developed along the water mass border in the water body with higher absorption (Fig. 2a). The RHR differences became more pronounced at greater depths. If, in turn, the water masses changed their locations, then the zone of lower RHR values occurred in the water of lower absorption (Fig. 2b). It should be noted that in the absence of external sources of radiant energy, the absorption decrease in a vertical direction cannot cause the occurrence of a local RHR minimum, because the total energy flux is decreasing with depth (Mobley 1994). The obtained non-monotonic distributions of RHR resulted from variability in the horizontal component of radiant energy flux along the  $x$  direction, since, in the above example, the vertical part of the flux was not affected by the absorption change at a given  $x$ . The strongest effects were expected to take place when radiant energy passed across the plane of sharp changes in the absorption value in a direction perpendicular to this plane. In other words, vector  $\text{grad}(a)$  should be parallel to the direction of solar rays. Therefore, it was supposed that  $g = |\mu_z| \mu_x^{-1} = 1.6$  and  $d=1.6$  in order to maintain an unchanged value of the absorption gradient in the middle of the mixing zone. Clearer patterns of local intensification of water warming or cooling were observed in the results (Fig. 2c, d). The zones of different RHR also extended down the border between the water masses, but included a greater area than in the previous case. The maxima with respect to the horizontal direction occurred in the subsurface water layer, and at a depth of 1 m were already developed (Fig. 3a). When absorption increased along vertical or horizontal directions separately, this could occur only at greater depths (Fig. 3a, b). At the maxima, RHR values were approximately one and a half times greater than values in the neighbouring area. In the next calculations, the condition on the absorption gradient was not as severe and  $d$  was set at 1.5 [m<sup>-1</sup>] and for the inclined border between water masses,  $d = 0.8$  [m<sup>-1</sup>]. In this case, the vertical absorption increase was too weak in comparison to the irradiance decrease with depth to induce an RHR maximum (Fig. 3c). In contrast, horizontal changes of absorption still generated RHR extrema. These disappeared when the  $d$



**Fig. 2.** Spatial distribution of the radiant heating rate (RHR) at the border between two water masses with different absorption coefficients. Rays of sunlight in the water propagate from left to right at an angle of  $32^\circ$  with respect to the vertical. Dashed lines show the centre of the mixing zone.

parameter was lower than approximately  $0.5 \text{ [m}^{-1}\text{]}$ . Similar spatial behaviour was displayed by a heating rate minimum occurring under an inverse absorption distribution; however, in this case RHR differences were lower (Fig. 3d).



**Fig. 3.** Comparison of the RHR horizontal and vertical profiles in an area of a water absorption spatial changes occurring between seawater masses with lower ( $a_1$ ) and higher ( $a_2$ ) absorption coefficients.

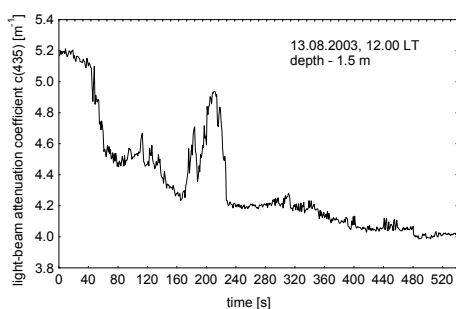
## DISCUSSION

Seawater backscatter strongly affects the magnitude of upwelling irradiance in the visible wave band. This effect was omitted when setting the border condition because the upwelling irradiance usually constitutes a small fraction of the total irradiance in the sea. At higher sun elevations it is usually of the order of 1% (Dera 2003). Based on numerical simulations of radiative transfer, it was also recognised that phytoplankton light attenuation alters the RHR, mainly through its influence on the irradiance transmission down the water column and not through its influence on the sea reflectance (Ohlmann et al. 2000). In the formulation of the equation for the spectral scalar irradiance, seawater light scattering was not taken into account. The varying relationship of absorption to scattering alters the mean direction of light propagation starting just below the sea surface (Bannister 1992, Berwald et al. 1995). A spatially variable scattering coefficient could have been introduced into Eq. (4) through

its relationship with the average cosines ( $\mu_x, \mu_z$ ). However, due to a lack of research results concerning horizontal optical variability in the sea, it was assumed that  $\mu_x, \mu_z$  did not change, as the relationship between scattering and absorption remained unchanged. The concept of the constancy of the light field directional distribution was successfully applied in estimating the irradiance attenuation coefficient in the depth range where the change of  $\mu_z$  was considered negligibly small in comparison to irradiance change (Aas 1981). In the present analysis, it was simply extended with respect to the horizontal direction.

In spite of all the simplifications that were made, the obtained results appear to be reasonable. The variation of the heating rate resulting from the changes in water absorption is consistent with the radiant power conservation law. It can be expected that additional light scattering variations will not alter the obtained water heating patterns provided they do not violate the rate of net irradiance changes. Stavn (1982) suggested that the increase of the scattering coefficient in a nepheloid (turbid) water layer makes the radiant heating higher because of the increase in the mean light path (reciprocal of  $\mu_z$ ), which enhances the absorption of light energy. However, the above result came from an approach using only vertical variability.

The main conclusion is that the combination of water absorption changes in the horizontal and vertical directions can be very effective in local water warming. The horizontal component of the absorption gradient can also lead to a decrease of RHR values. It should be noted that over several hours on a sunny



**Fig. 4.** Exemplary time series of the blue light-beam attenuation coefficient in coastal waters (Gulf of Gdańsk, Baltic Sea) measured during the drift of a ship with a velocity 0.3 - 0.6 [knots], (based on author's own field research results).

day in the spring or summer, such RHR effects can result in detectable variations in temperature in the sea surface layer. An important question is whether sufficiently high horizontal gradients of optical properties occur in natural waters. Such an occurrence is very probable, because a time series of the blue light attenuation coefficient (sum of the absorption and scattering coefficient) measured at a constant depth in coastal waters showed significant variability at the borders of the patchy structures (Fig. 4).

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